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Physics and Mathematics in Quantum Stochastic Process

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Abstract

It is shown that the time-evolution of a dissipative system can be interpreted as a traverse of the system in a set of the unitarily inequivalent representation spaces. It is also shown that there exists uncountable number of different descriptions of the system of quantum differential equations, and that the physical meaning of the different descriptions can be attributed to how much one renormalized the line-width in an energy spectrum caused by uncommutative part of a random force operator.

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量子確率過程の物理と数学

(Physics and Mathematics in Quantum Stochastic Process)

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1 Introduction

Recently we succeeded to construct a unified framework of the *canonical operator formalism* for quantum stochastic differential equations within Non-Equilibrium Thermo Field Dynamics (NETFD) [1]-[9] for the first time to put all the formulations of stochastic differential equations for quantum systems, i.e., the Langevin equation and the stochastic Liouville equation [10] together with corresponding quantum master equation, into a unified method (see Fig. 1). It was possible only within the formalism of NETFD.

In this paper, we will show that the time evolution of a dissipative system can be interpreted as a traverse of the system in a set of the unitarily inequivalent representation spaces. We believe that the set constitutes a measured space which corresponds to the Γ phase-space of classical statistical mechanics. We will also show that there exists uncountable number of different descriptions of the system of quantum differential equations, and that the physical meaning of the different descriptions can be attributed to how much one renormalized the line-width in an energy spectrum caused by uncommutative effects of a random force operator.

We will treat in this paper a non-stationary system of a stochastic semi-free particles. The hat-Hamiltonian for the *stochastic semi-free* field is bi-linear in a, $a^\dagger$, $dF(t)$, $dF^\dagger(t)$ and their tilde conjugates, and is invariant under the phase transformation $a \to a e^{i\theta}$, and $dF(t) \to dF(t)\ e^{i\theta}$. Here, a, $a^\dagger$ and their tilde conjugates are stochastic operators of a relevant system satisfying the canonical commutation relation

$$[a,\ a^\dagger] = 1, \qquad [\tilde{a},\ \tilde{a}^\dagger] = 1, \tag{1}$$

whereas $dF(t)$, $dF^\dagger(t)$ and their conjugates are random force operators. The tilde and non-tilde operators are related with each other by the relations

$$\langle 1|a^\dagger = \langle 1|\tilde{a}, \tag{2}$$
$$\langle|dF^\dagger(t) = \langle|d\tilde{F}(t), \tag{3}$$

where $\langle 1|$ and $\langle|$ are respectively the thermal bra-vacuum of the relevant system and of the random force.

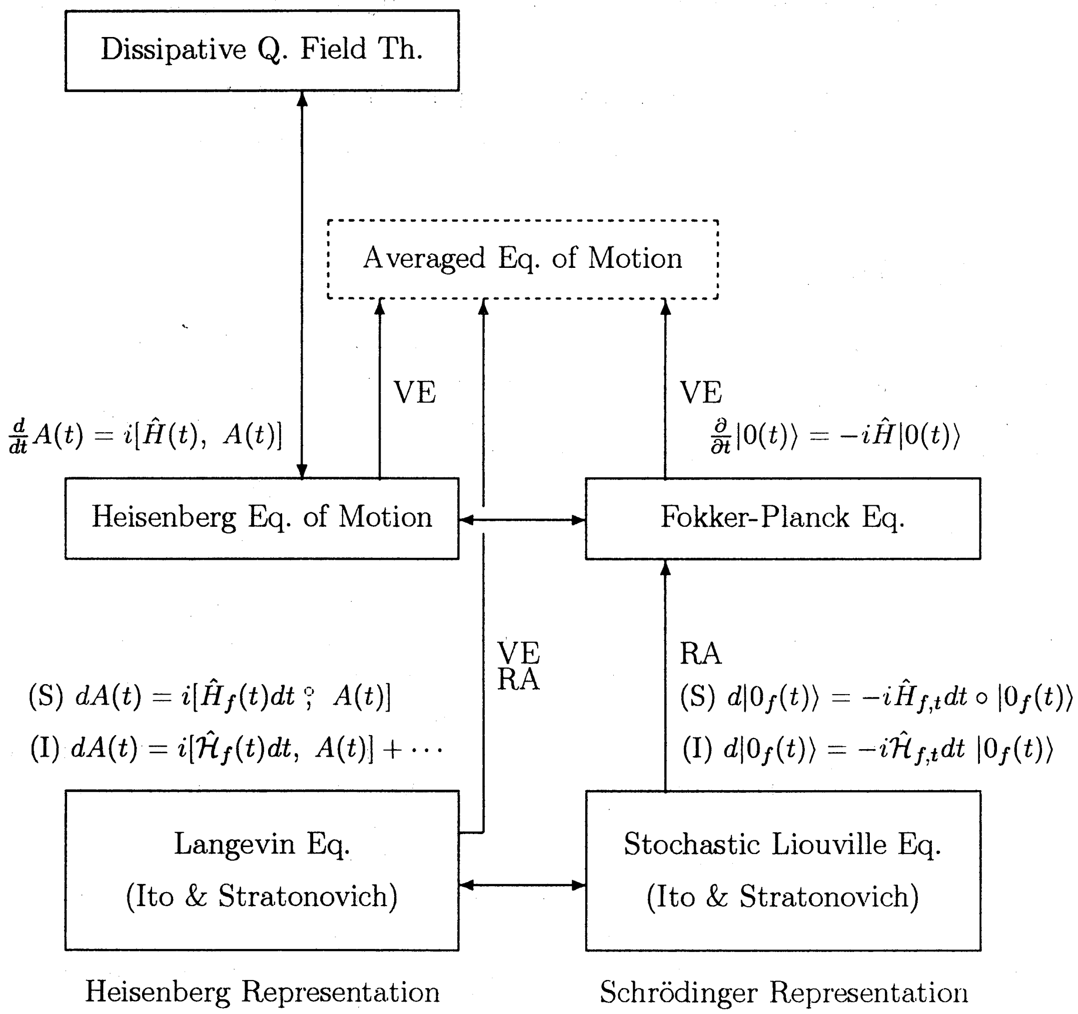

Figure 1: Structure of the Formalism. RA stands for the random average. VE stands for the vacuum expectation. (I) and (S) indicate Ito and Stratonovich types, respectively.

The *tilde conjugation* $\sim$ is defined by:

$$(A_1 A_2)^{\sim} = \tilde{A}_1 \tilde{A}_2, \tag{4}$$
$$(c_1 A_1 + c_2 A_2)^{\sim} = c_1^* \tilde{A}_1 + c_2^* \tilde{A}_2, \tag{5}$$
$$(\tilde{A})^{\sim} = A, \tag{6}$$
$$(A^\dagger)^{\sim} = \tilde{A}^\dagger, \tag{7}$$

where c_1 and c_2 are c-numbers. Any operator A in NETFD is accompanied by its partner (tilde) operator $\tilde{A}$.

2 Representation Space of Random Force Operators

2.1 Fock's Space

We take the vectors:

$$|t_1, t_2, \cdots, t_n\rangle = \frac{1}{\sqrt{n!}} b^\dagger(t_1) b^\dagger(t_2) \cdots b^\dagger(t_n)|0\rangle, \tag{8}$$

as a set of bases for a Fock space. The argument t represents time. The vacuum $|0\rangle$ is defined by

$$b(t)|0\rangle = 0. \tag{9}$$

The annihilation and creation operators $b(t)$, $b^\dagger(t)$ satisfy the canonical commutation relation:

$$[b(t),\ b^\dagger(t')] = \delta(t - t'). \tag{10}$$

The bases form an ortho-normal and complete set:

$$\langle t_1, \cdots, t_n | t_1', \cdots, t_m' \rangle = \delta_{n,m} \frac{1}{n!} \sum_{(P)} \delta(t_1 - t_1') \cdots \delta(t_n - t_n'), \tag{11}$$

$$\sum_{n=0}^{\infty} \left(\prod_{\ell=1}^{n} \int_0^\infty dt_\ell \right) |t_1, \cdots, t_n\rangle\langle t_1, \cdots, t_n| = I. \tag{12}$$

The Fock space $\Gamma(\mathcal{H})$ over a Hilbert space $\mathcal{H}$ is the infinite Hilbert space direct sum $\Gamma(\mathcal{H}) = \bigoplus_{n=0}^{\infty} \mathcal{H}^{\hat{\otimes} n}$, where $\mathcal{H}^{\hat{\otimes} n=0} = \boldsymbol{C}$, and, for $n \geq 1$, $\mathcal{H}^{\hat{\otimes} n}$ is the symmetric subspace of the n-fold Hilbert space tensor product of $\mathcal{H}$ (the Wiener-Ito expansion).

For $|\psi\rangle \in \Gamma(\mathcal{H})$, we have

$$|\psi\rangle = \sum_{n=0}^{\infty} \left(\prod_{\ell=1}^{n} \int_0^\infty dt_\ell \right) |t_1, \cdots, t_n\rangle \psi_n(t_1, \cdots, t_n), \tag{13}$$

where $\psi_n(t_1, \cdots, t_n) = \langle t_1, \cdots, t_n|\psi\rangle \in \mathcal{H}^{\hat{\oplus} n}$. This situation is similar to the one in quantum field theory when expanding a state in a Fock space in terms of the state vectors in the n-particle subspace. In that case, ψ_n is the wave-function of n-particle system in quantum mechanics.

2.2 Quantum Brownian Motion

Introducing the operators

$$B(t) = \int_0^t dt'\ b(t'), \qquad B^\dagger(t) = \int_0^t dt'\ b^\dagger(t'), \tag{14}$$

for $t \geq 0$, we see that they satisfy

$$B(0) = 0, \qquad [B(s),\ B^\dagger(t)] = \min(s,t). \tag{15}$$

This shows that $B(t)$ and $B^\dagger(t)$ are the operators representing quantum Brownian motion [11, 12].

The definition and the existence of the operators $b(t)$ and $b^\dagger(t)$ are guaranteed by Hida and Obata [13, 14].

2.3 Ito's Stochastic Product

When a stochastic integral $I(t)$:

$$I(t) = \int_0^t dt' \left\{ dB^\dagger(t')F(t') + G(t')dB(t') + H(t')dt' \right\}, \tag{16}$$

with $t \geq 0$ exists, it can be written in a differential form

$$dI(t) = dB^\dagger(t)F(t) + G(t)dB(t) + H(t)dt, \qquad I(0) = 0. \tag{17}$$

For

$$dI_i(t) = dB^\dagger(t)F_i(t) + G_i(t)dB(t) + H_i(t)dt, \qquad I_i(0) = 0, \tag{18}$$

$(i = 1, 2)$, we have the Ito stochastic product [15]

$$\begin{aligned} d(I_1 I_2) &= dB^\dagger \left(F_1 I_2 + I_1 F_2\right) + \left(G_1 I_2 + I_1 G_2\right) dB(t) + \left(H_1 I_2 + I_1 H_2 + G_1 F_2\right) dt \\ &= dI_1 I_2 + I_1 dI_2 + dI_1 dI_2. \end{aligned} \tag{19}$$

Here we used

$$dB(t)dB^\dagger(t) = dt, \tag{20}$$

which can be shown by the commutation relation $[dB(t),\ dB^\dagger(t)] = dt$. Note that the commutation relation is a consequence of (15). In precise, the Ito formula (19) is proven in the representation of the exponential vectors.

2.4 Thermal Space

Now, put the above materials in the Hilbert space into the thermal space within NETFD. The approach seems somewhat related to the one by [16].

The operators representing the quantum Brownian motion annihilate the vacuums $|0\rangle$ and $\langle 0|$:

$$dB(t)|0\rangle = 0, \qquad d\tilde{B}(t)|0\rangle = 0, \qquad \langle 0|dB^\dagger(t) = 0, \qquad \langle 0|d\tilde{B}^\dagger(t) = 0. \tag{21}$$

Let us introduce a set of new operators by the relation

$$d\mathcal{B}(t)^{\mu} = \bar{B}(t)^{\mu\nu} dB(t)^{\nu}, \tag{22}$$

with the Bogoliubov transformation defined by

$$\bar{B}(t)^{\mu\nu} = \begin{pmatrix} n(t) + 1 + (2\kappa(t))^{-1} dn(t)/dt, & -n(t) - (2\kappa(t))^{-1} dn(t)/dt \\ -1, & 1 \end{pmatrix}, \tag{23}$$

where the one particle distribution function $n(t)$ is specified by the Boltzmann equation

$$\frac{d}{dt} n(t) = -2\kappa(t) n(t) + i\Sigma^{<}(t). \tag{24}$$

The function $\Sigma^{<}(t)$ is given when the interaction hat-Hamiltonian is specified. We introduced the thermal doublet:

$$dB(t)^{\mu=1} = dB(t), \quad dB(t)^{\mu=2} = d\tilde{B}^{\dagger}(t), \quad d\bar{B}(t)^{\mu=1} = dB^{\dagger}(t), \quad d\bar{B}(t)^{\mu=2} = -d\tilde{B}(t), \tag{25}$$

and the similar doublet notations for $d\mathcal{B}(t)^{\mu}$ and $d\bar{\mathcal{B}}(t)^{\mu}$. The new operators annihilate the new vacuums $\langle|$ and $|\rangle$:

$$d\mathcal{B}(t)|\rangle = 0, \qquad d\tilde{\mathcal{B}}(t)|\rangle = 0, \qquad \langle|d\mathcal{B}^{\dagger}(t) = 0, \qquad \langle|d\tilde{\mathcal{B}}^{\dagger}(t) = 0. \tag{26}$$

2.5 Unitary Inequivalence

The generator $\hat{U}$ inducing the Bogoliubov transformation (23) in the form

$$d\mathcal{B}(t)^{\mu} = \hat{U}^{-1} dB(t)^{\mu} \hat{U}, \tag{27}$$

is given by

$$\hat{U} = \exp\left[-\int_0^{\infty} dt \left(n(t) + \frac{1}{2\kappa(t)} \frac{dn(t)}{dt}\right) b^{\dagger}(t)\tilde{b}^{\dagger}(t)\right] \exp\left[\int_0^{\infty} dt b(t)\tilde{b}(t)\right]. \tag{28}$$

Then, we see formally that

$$\begin{aligned} |\rangle &= \hat{U}^{-1}|0\rangle \\ &= \exp\left[-\delta(0) \int_0^{\infty} dt \ln\left(1 + \frac{i\Sigma^{<}(t)}{2\kappa(t)}\right)\right] \\ &\quad \exp\left[\int_0^{\infty} dt \frac{i\Sigma^{<}(t)}{2\kappa(t) + i\Sigma^{<}(t)} b^{\dagger}(t)\tilde{b}^{\dagger}(t)\right] |0\rangle. \end{aligned} \tag{29}$$

It shows the unitary inequivalence of the vacuums $|\rangle$ and $|0\rangle$.

The vacuum $|\rangle$ and $\langle|$ can be decomposed into an infinite direct product of unitarily inequivalent vacuums:

$$
\begin{aligned}
|\rangle &= \hat{U}^{-1}|0\rangle \\
&= \prod_{t=0}^{\infty} \exp\left[-\delta(0)\int_t^{t+dt} dt' \ln\left(1+\frac{i\Sigma^<(t')}{2\kappa(t')}\right)\right] \\
&\quad \exp\left[\int_t^{t+dt} dt' \frac{i\Sigma^<(t')}{2\kappa(t')+i\Sigma^<(t')} b^\dagger(t')\tilde{b}^\dagger(t')\right]|0\rangle \\
&= \prod_{t=0}^{\infty} |t, t+dt\rangle,
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
\langle| &= \langle 0|\hat{U} \\
&= \prod_{t=0}^{\infty} \langle 0|\exp\left[\int_t^{t+dt} dt' b(t')\tilde{b}(t')\right] \\
&= \prod_{t=0}^{\infty} \langle t, t+dt|.
\end{aligned}
\tag{31}
$$

We see that

$$\langle t, t+dt|t, t+dt\rangle = 1, \tag{32}$$

$$\langle t', t'+dt|t, t+dt\rangle = \exp\left[-\delta(0)\int_t^{t+dt} dt'' \ln\left(1+\frac{i\Sigma^<(t'')}{2\kappa(t'')}\right)\right], \tag{33}$$

for $t \neq t'$. The last equation (33) indicates the unitary inequivalence between the Fock's spaces labeled $t, t+dt$ and $t', t'+dt$.

2.6 Random Force Operators

In the following, we will use the representation space constructed on the vacuums $\langle|$ and $|\rangle$. Then, we have, for example,

$$
\begin{aligned}
\langle|dB^\dagger(t)dB(t)|\rangle &= \left(n(t) + \frac{1}{2\kappa(t)}\frac{dn(t)}{dt}\right) dt, \\
\langle|dB(t)dB^\dagger(t)|\rangle &= \left(n(t) + 1 + \frac{1}{2\kappa(t)}\frac{dn(t)}{dt}\right) dt,
\end{aligned}
\tag{34}
$$

which was derived by inspecting $\langle|d\tilde{B}(t)dB(t)|\rangle$ with the help of the thermal state conditions (26).

For a practical convenience, we introduce the random force operators by

$$dF(t) = \sqrt{2\kappa(t)}\, dB(t), \qquad dF^\dagger(t) = \sqrt{2\kappa(t)}\, dB^\dagger(t). \tag{35}$$

Then, we have $\left\langle dF(t)\right\rangle = \left\langle d\tilde{F}(t)\right\rangle = \left\langle dF^\dagger(t)\right\rangle = \left\langle d\tilde{F}^\dagger(t)\right\rangle = 0$, and

$$\begin{aligned}\left\langle dF^\dagger(t)dF(s)\right\rangle &= \left(2\kappa(t)n(t) + \frac{dn(t)}{dt}\right)\delta(t-s)dtds,\\ \left\langle dF(t)dF^\dagger(s)\right\rangle &= \left(2\kappa(t)\left(n(t)+1\right) + \frac{dn(t)}{dt}\right)\delta(t-s)dtds,\end{aligned} \tag{36}$$

and zero for other combinations (see (34)). Here we introduced an abbreviation $\langle\cdots\rangle = \langle|\cdots|\rangle$.

The thermal state condition (26) reads

$$\left(1 + n(t) + \frac{1}{2\kappa(t)}\frac{dn(t)}{dt}\right) dF(t)|\rangle = \left(n(t) + \frac{1}{2\kappa(t)}\frac{dn(t)}{dt}\right) d\tilde{F}^\dagger(t)|\rangle, \tag{37}$$

and (3).

3 Stochastic Semi-Free System

3.1 Model

A *non-stationary* stochastic semi-free system (a stochastic model of a damped harmonic oscillator) is specified by the stochastic Liouville equation of Stratonovich type:

$$d|0_f(t)\rangle = -i\hat{H}_{f,t}dt \circ |0_f(t)\rangle, \tag{38}$$

with the stochastic hat-Hamiltonian

$$\begin{aligned}\hat{H}_{f,t}dt &= \hat{H}_{S,t}dt + i\hat{\Pi}_{R,t}dt + d\hat{M}_t &(39)\\ &= \hat{H}_{S,t}dt + \left[\alpha^{\maltese}\left(id\alpha + [\hat{H}_{S,t}dt,\ \alpha]\right) - \text{t.c.}\right], &(40)\end{aligned}$$

where the operator $\hat{\Pi}_{R,t}$ representing a relaxation effect, the martingale $d\hat{M}_t$ and the flow operators $d\alpha$, $d\tilde{\alpha}$ are specified, respectively, by

$$\hat{\Pi}_{R,t} = -\kappa(t)\left(\alpha^{\maltese}\alpha + \text{t.c.}\right), \tag{41}$$

$$d\hat{M}_t = i\left(\alpha^{\maltese}dW(t) + \text{t.c.}\right), \tag{42}$$

$$d\alpha = i[\hat{H}_{S,t}dt,\ \alpha] - \kappa(t)\alpha dt + dW(t), \tag{43}$$

and its tilde conjugate.

We introduced a set of canonical stochastic operators

$$\alpha = \mu a + \nu\tilde{a}^\dagger, \qquad \alpha^{\maltese} = a^\dagger - \tilde{a}, \tag{44}$$

with $\mu + \nu = 1$, which satisfy the commutation relation

$$[\alpha, \ \alpha^{\dagger}] = 1. \tag{45}$$

The random force operators $dW(t), d\tilde{W}(t)$ are of the quantum stochastic Wiener process satisfying

$$\begin{aligned}
\left\langle dW(t)\right\rangle &= \left\langle d\tilde{W}(t)\right\rangle = 0, &(46)\\
\left\langle dW(t)dW(s)\right\rangle &= \left\langle d\tilde{W}(t)d\tilde{W}(s)\right\rangle = 0, &(47)\\
\left\langle dW(t)d\tilde{W}(s)\right\rangle &= \left\langle d\tilde{W}(s)dW(t)\right\rangle \\
&= \left(2\kappa(t)\left(n(t)+\nu\right) + \frac{dn(t)}{dt}\right)\delta(t-s)dtds, &(48)
\end{aligned}$$

where the random force operator $dW(t)$ is defined by

$$dW(t) = \mu dF(t) + \nu d\tilde{F}^{\dagger}(t), \tag{49}$$

with $\mu + \nu = 1$. The ***original*** random force operators $dF(t)$ and $dF^{\dagger}(t)$ are of the ***non-stationary*** Gaussian white process derived in the previous section.

Within the stochastic convergence, these correlations reduce to[1]

$$\begin{aligned}
dW(t) &= d\tilde{W}(t) = 0, &(51)\\
dW(t)dW(s) &= d\tilde{W}(t)d\tilde{W}(s) = 0, &(52)\\
dW(t)d\tilde{W}(s) &= d\tilde{W}(s)dW(t) \\
&= \left(2\kappa(t)\left(n(t)+\nu\right) + \frac{dn(t)}{dt}\right)\delta(t-s)dtds \\
&= \left(i\Sigma^{<}(t) + 2\nu\kappa(t)\right)\delta(t-s)dtds. &(53)
\end{aligned}$$

We introduced the symbol $\circ$ in order to indicate the Stratonovich stochastic multiplication [17].

The quantum stochastic Liouville equation (38) preserves the characteristics of the stochastic Liouville equation [10] of classical systems, i.e., the stochastic distribution function satisfies the conservation of probability within the phase space of a relevant system. This means in NETFD that

$$\langle 1|0_f(t)\rangle = 1, \tag{54}$$

leading to

$$\langle 1|\hat{H}_{f,t}dt = 0. \tag{55}$$

Here the thermal bra-vacuum $\langle 1|$ is of the relevant system.

[1]For equal time $t = s$, (53) reads

$$dW(t)d\tilde{W}(t) = d\tilde{W}(t)dW(t) = \left(i\Sigma^{<}(t) + 2\nu\kappa(t)\right)dt. \tag{50}$$

3.2 Quantum Langevin Equations

For the dynamical quantity $A(t)$ of the relevant system, the quantum Langevin equation of the Stratonovich type is given by the stochastic Heisenberg equation as [4]

$$dA(t) = i[\hat{H}_f(t)dt \,♀\, A(t)] \tag{56}$$

$$\begin{aligned} &= i[\hat{H}_S(t),\ A(t)]dt \\ &\quad +\kappa(t)\left\{[\alpha^{♀}(t)\alpha(t),\ A(t)] + [\tilde{\alpha}^{♀}(t)\tilde{\alpha}(t),\ A(t)]\right\} dt \\ &\quad -\left\{[\alpha^{♀}(t),\ A(t)] \circ dW(t) + [\tilde{\alpha}^{♀}(t),\ A(t)] \circ d\tilde{W}(t)\right\}. \end{aligned} \tag{57}$$

3.3 Solving the Stochastic Liouville Equation

The quantum stochastic Liouville equation of the present system in the Ito type expression is given by

$$d|0_f(t)\rangle = -i\hat{\mathcal{H}}_{f,t}dt\ |0_f(t)\rangle, \tag{58}$$

with

$$\hat{\mathcal{H}}_{f,t}dt = \hat{H}_t dt + d\hat{M}_t, \tag{59}$$

where $\hat{H}_t$ is given by

$$\hat{H}_t = \hat{H}_{S,t} + i\hat{\Pi}_t. \tag{60}$$

Here, $\hat{\Pi}_t$ is defined by

$$\hat{\Pi}_t = \hat{\Pi}_{R,t} + \hat{\Pi}_{D,t}, \tag{61}$$

with

$$\hat{\Pi}_{D,t} = 2\left(\kappa(t)\,(n(t)+\nu) + \frac{dn(t)}{dt}\right)\alpha^{♀}\tilde{\alpha}^{♀}. \tag{62}$$

The diffusive time-evolution operator $\hat{\Pi}_{D,t}$ contains the information how much the unitarily inequivalent Fock's spaces for the random force operators overlaps with each other in the time axis.

Note that the orthogonality

$$\langle|d\hat{M}_t|0_f(t)\rangle = 0. \tag{63}$$

3.4 Fokker-Planck Equation

Taking the random average of the stochastic Liouville equation (58), we obtain the Fokker-Planck equation

$$\frac{\partial}{\partial t}|0(t)\rangle = -i\hat{H}_t|0(t)\rangle, \tag{64}$$

with $|0(t)\rangle = \langle|0_f(t)\rangle$. It can be solved to give

$$|0(t)\rangle = \mathrm{e}^{[n(t)-n(0)]\gamma^{♀}\tilde{\gamma}^{♀}}|0(0)\rangle. \tag{65}$$

The creation operators $\gamma^{\dagger}$ and $\tilde{\gamma}^{\dagger}$ are defined through

$$\alpha^{\mu} = \begin{pmatrix} 1 & n(t)+\nu \\ 0 & 1 \end{pmatrix}^{\mu\nu} \gamma^{\nu}, \tag{66}$$

with the thermal doublet:

$$\gamma^{\mu=1} = \gamma_t, \quad \gamma^{\mu=2} = \tilde{\gamma}^{\dagger}, \quad \bar{\gamma}^{\mu=1} = \gamma^{\dagger}, \quad \bar{\gamma}^{\mu=2} = -\tilde{\gamma}_t, \tag{67}$$

and the similar definition for α^{μ}. These creation and annihilation operators annihilate the vacuums:

$$\gamma_t|0(t)\rangle = 0, \quad \tilde{\gamma}_t|0(t)\rangle = 0, \quad \langle 1|\gamma^{\dagger} = 0, \quad \langle 1|\tilde{\gamma}^{\dagger} = 0. \tag{68}$$

The solution (65) of the Fokker-Planck equation shows that the dissipative time-evolution of the relevant system can be interpreted as a condensation of $\gamma^{\dagger}\tilde{\gamma}^{\dagger}$-pairs into the thermal vacuum.

4 Renormalization of the Uncommutative Part of the Random Force Operators

We introduce here the generalized stochastic hat-Hamiltonian of the Stratonovich type by

$$\hat{H}^{\lambda}_{f,t}dt = \hat{H}_{S,t}dt + i\lambda\hat{\Pi}_{R,t}dt + d\hat{M}^{\lambda}_t, \tag{69}$$

with

$$d\hat{M}^{\lambda}_t = i\left\{\left[\alpha^{\dagger}dW(t) + \text{t.c.}\right] - (1-\lambda)\left[\alpha dW^{\dagger}(t) + \text{t.c.}\right]\right\}, \tag{70}$$

where λ is a real number satisfying $0 \le \lambda \le 1$.

In addition to the random force operators $dW(t)$ and its tilde conjugate, we need to introduce

$$dW^{\dagger}(t) = dF^{\dagger}(t) - d\tilde{F}(t), \tag{71}$$

and its tilde conjugate which annihilate the ket-vacuum $\langle|$:

$$\langle|dW^{\dagger}(t) = 0, \qquad \langle|d\tilde{W}^{\dagger}(t) = 0. \tag{72}$$

The additional random force operators satisfy

$$dW^{\dagger}(t) = d\tilde{W}^{\dagger}(t) = 0, \qquad dW^{\dagger}(t)dW(s) = d\tilde{W}^{\dagger}(t)d\tilde{W}(s) = 0, \tag{73}$$

$$dW(t)dW^{\dagger}(s) = d\tilde{W}(t)d\tilde{W}^{\dagger}(s) = 2\kappa(t)\delta(t-s)dt\, ds, \tag{74}$$

within the stochastic convergence.

In the generalized description, the conservation of the probability is satisfied in the form:

$$\langle\langle 1|0_f(t)\rangle\rangle = 1, \tag{75}$$

where $\langle\langle 1| = \langle|\langle 1|$.

We can show that the stochastic hat-Hamiltonian of the Ito type reduces to

$$\hat{\mathcal{H}}_{f,t}dt = \hat{H}_t dt + d\hat{M}_t^{\lambda}, \tag{76}$$

(c.f., (59)). Therefore, the Fokker-Planck equation remains the same as (64).

When $\lambda = 1$, the random force operators become commutative, leading to the system given in the previous section. On the other hand, when $\lambda = 0$, the generalized hat-Hamiltonian (69) becomes hermitian. This version is intimately related to the approaches performed by mathematicians [11, 12, 16, 18] based on the stochastic Schrödinger equation (see also [19, 20]). For the intermediate λ, the relaxation rate function in $\lambda\hat{\Pi}_{R,t}$ is partially *renormalized*, i.e., $\lambda\kappa(t)$, within the Stratonovich description. Within the Ito description, the relaxation rate function is fully renormalized in the stochastic hat-Hamiltonian of the Ito type.

We can interpret that the translation to the Ito description is to orthogonalize the martingale part to the thermal vacuum, and to renormalize the spectrum of the semi-free particle to have an observable (physical) line-width.

5 Summary

We showed that the time evolution of a dissipative system can be interpreted as a traverse of the system in a set of the unitarily inequivalent representation spaces. We believe that the set constitutes a measured space which corresponds to the Γ phase-space of classical statistical mechanics. Now, we are trying to input a measure into the space which may provide us with a new concept of *entropy*.

We also showed that there exists uncountable number of different descriptions of the system of quantum differential equations, and that the physical meaning of the different descriptions can be attributed to how much one renormalized the line-width in an energy spectrum caused by uncommutative effects of a random force operator. We are investigating the deeper meaning of the renormalization within the present new context which was revealed only by the formalism of NETFD.

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